Thermodynamics and hydrodynamics beyond strong coupling

(lecture 3)

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Outline:

- 't Hooft coupling and finite-N corrections
- Violation of the shear viscosity bound
- Causality constraints and hydrodynamics

(from lecture 2)

Similar analysis can be performed in generic (infinitely) strongly coupled (planar) gauge theory

$$\frac{1}{N g_{YM}^2} \to 0 \qquad \& \qquad N \to \infty \qquad (\text{with } N g_{YM}^2 \to \text{const })$$

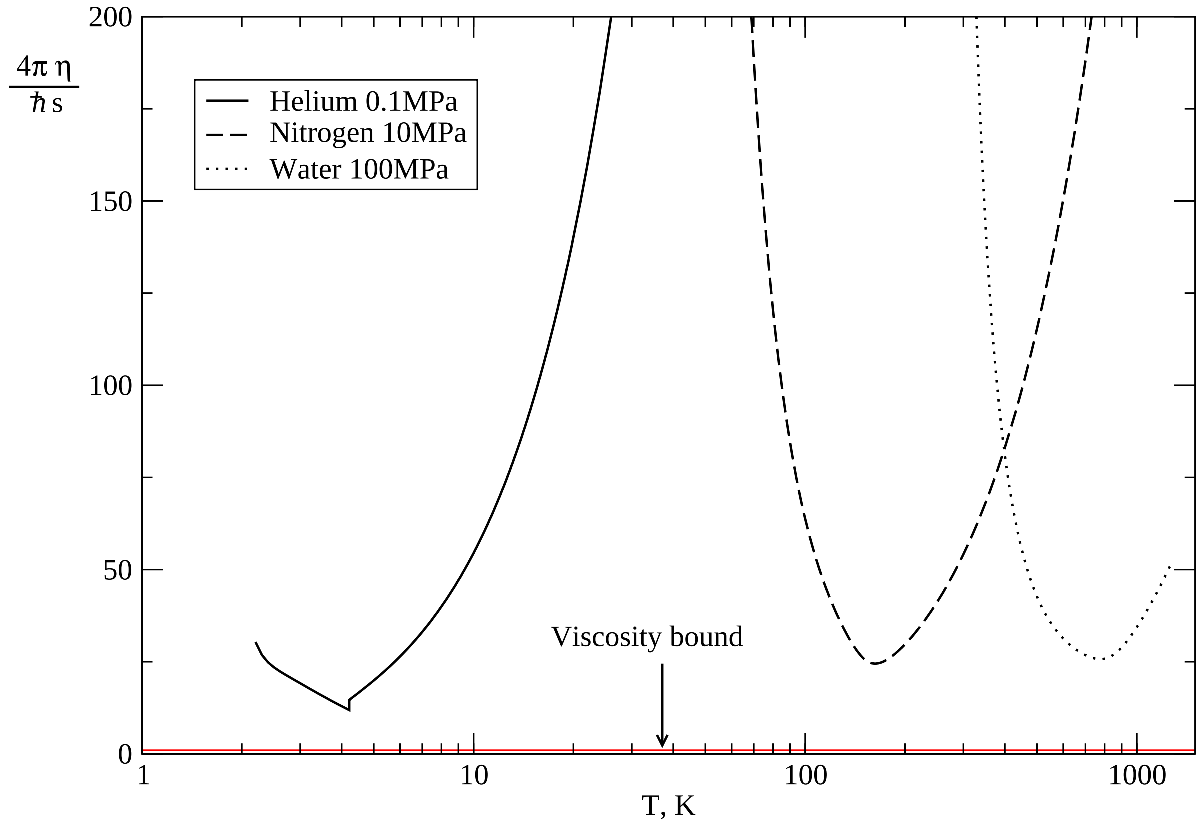
We find:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

for:

- any product of gauge groups and matter content in arbitrary representations
- arbitrary non-conformal deformations (masses, non-zero β-functions)
- arbitrary chemical potentials for conserved $U(1)$'s
- non-commutativity of the background of space-time
- presence of (scalar) Goldstone modes from spontaneous breaking of continuous symmetries (superfluid/superconductor)
- background electromagnetic fields

KSS viscosity bound



From: P.Kovtun, D.T.Son, A.O.Starinets, Phys.Rev.Lett. 94 (2005) 111601

What can save us from the shear viscosity universality?

Recall basic AdS/CFT correspondence:

gauge theory		string theory
$\mathcal{N}=4$ $SU(N)$ SYM	$\Longleftrightarrow$	N-units of 5-form flux in type IIB string theory
g_{YM}^2	$\Longleftrightarrow$	g_s

$\Longrightarrow$ Each of the duality frames are valid in complimentary regimes. In the 't Hooft limit (planar limit), $N \to \infty$, $g_{YM}^2 \to 0$ with Ng_{YM}^2 kept fixed:

- for $g_{YM}^2 N \ll 1$ we can use a standard perturbation theory
- for $g_{YM}^2 N \ll 1$ we can use effective supergravity description of type IIB string theory on $AdS_5 \times S^5$

$\Longrightarrow$ In the above regime we can incorporate corrections:

$$\frac{1}{N}\text{-corrections} \quad \Longleftrightarrow \quad g_s\text{-corrections}$$

$$\frac{1}{Ng_{YM}^2}\text{-corrections} \quad \Longleftrightarrow \quad \alpha'\text{-corrections}$$

Realistically, we can compute (reliably) only:

- leading order 't Hooft coupling corrections
- and only for $\mathcal{N} = 4$ SYM

$\Longrightarrow$ We can not do it for:

- nonconformal backgrounds as they are sourced by RR fluxes, and even though partial results for α' corrected type IIb SUGRA in the presence of RR fluxes is known, this is not enough
- we do not know how to reliably compute quantum g_s corrections in backgrounds with RR fluxes (some partial results are available though)

$\Longrightarrow$ The implications of the above are that the String Theory

- <u>can</u> probe if the KSS viscosity bound is violated (violation can be to leading order beyond SUGRA approximation)
- <u>can not</u> tell what and (whether it exists) any new bound on $\frac{\eta}{s}$

Beyond infinite 't Hooft coupling for η/s in AdS/CFT

$\Longrightarrow$ Consider $\mathcal{N}=4$ $SU(N)$ SYM in the planar ('t Hooft limit):

$$N \to \infty\,, \qquad g_{YM}^2 \to 0 \qquad \Rightarrow \lambda \equiv N g_{YM}^2 = \text{const}$$

$\Longrightarrow$ We would like to understand leading in $\frac{1}{\lambda}$ corrections to the (first-order) transport coefficients

- first, since the theory is conformal for all values of λ,

$$c_s^2 = \frac{1}{3}\,, \qquad \zeta = 0$$

and the only corrections can happen for the shear viscosity

- It can be derived from the string theory that in the planar limit, the leading $\frac{1}{\lambda}$ corrections for *any conformal plasma* (with equal central charges — see later) are described by the following effective action

$$S_5 = \frac{1}{16\pi G_5} \int d^5\xi \sqrt{-g}\left(R + 12 + \gamma W + \mathcal{O}(\gamma^2)\right)$$

$$W \equiv C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} + \frac{1}{2} C^{hkmn} C_{rqmn} C_h{}^{rsp} C^q{}_{rsk}$$

where C^{hmnk} is a 5d Weyl tensor, and

$$\gamma = \frac{1}{8}\xi(3)(\alpha')^3 \qquad \Rightarrow \text{ (for } \mathcal{N}=4) \qquad \gamma = \frac{1}{8}\xi(3)\lambda^{-3/2}$$

$\Longrightarrow$ We can generalize the computation of the sound channel quasinormal mode and extract shear viscosity ratio from the sound attenuation

$$w = \pm\frac{1}{\sqrt{3}} - i\Gamma k^2 + \mathcal{O}(k^3)\,, \qquad 2\pi T\Gamma = \frac{1}{3}4\pi\frac{\eta}{s}$$

$\Longrightarrow$ I will outline the main steps of the computation

- First, one needs to determine the corrected equilibrium thermodynamics of the theory — it can be extracted from the α'-corrected black D3-brane solution:

$$P = \frac{1}{3}\mathcal{E} = \frac{\pi^2 N^2 T^4}{8}\ \left(1 + 15\gamma + \mathcal{O}(\gamma^2)\right)$$

■ Second, we need to derive the equations for the helicity-0 metric fluctuations to order $\mathcal{O}(\gamma)$ in the $\mathcal{O}(\gamma)$ corrected black-brane solution, and set-up the gauge-invariant combination of the fluctuations. We find:

- the following combination of fluctuations is gauge-invariant, and decouples

$$Z = 4\frac{q}{\omega}\ H_{tz} + 2\ H_{zz} - H_{aa}\left(1 - \frac{q^2}{\omega^2}\frac{c_1' c_1}{c_2' c_2}\right) + 2\frac{q^2}{\omega^2}\frac{c_1^2}{c_2^2}\ H_{tt}$$

where c_i are the $\mathcal{O}(g)$-correction metric warp factors

$$ds_5^2 = -c_1^2(r)\ dt^2 + c_2^2(r)\ d\vec{x}^2 + dr^2$$

- the EOM for Z takes the form

$$A\ Z'' + B\ Z' + C\ Z = \gamma\left(D\ Z^{IV} + E\ Z''' + F\ Z'' + G\ Z' + H\ Z\right) + \mathcal{O}(\gamma^2)$$

where we extracted explicit γ dependence

- boundary conditions on Z are unchanged:

$$Z(r \to 1) \sim (1-r)^{-i\mathfrak{w}/2}\,, \qquad Z(r \to 0) \sim r^2$$

where $\mathfrak{w} = w/(2\pi T)$ — it is important to include α' corrections to the BH temperature!

$\Longrightarrow$ At appears that equation for Z is forth order, while I'm setting only 2 boundary conditions — this is consistent as the higher-derivative effective action derived in string theory is consistent **only** perturbatively! It simply does not make sense (in a context of effective action to study the propagation of modes which appear non-perturbatively in γ).

$\Longrightarrow$ In fact, one must use lower order equations of motion for Z to eliminate the higher derivatives. Thus, using

$$A\ Z'' + B\ Z' + C\ Z = \mathcal{O}(\gamma^0)$$

we can eliminate all the derivatives on the RHS up to the first order:

$$A\ Z'' + B\ Z' + C\ Z = \gamma \left(\tilde{G}\ Z' + \tilde{H}\ Z\right) + \mathcal{O}(\gamma^2)$$

The boundary value problem for the quasinormal mode Z to order $\mathcal{G}(\gamma)$ is well defined

We find the following $\mathcal{O}(g)$ dispersion relation for the sound channel quasinormal mode:

$$\mathfrak{w} = \frac{1}{\sqrt{3}}\mathfrak{q} - i\mathfrak{q}^2\left(\frac{1}{3} + \frac{120}{3}\gamma\right) + \mathcal{O}(\mathfrak{q}^3)$$

which produces

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + 120\gamma + \mathcal{O}(\gamma^2)\right) = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \mathcal{O}(\lambda^{-3})\right)$$

$\Longrightarrow$ I can not cover it here, but it is possible to compute leading $\frac{1}{N}$ corrections to the shear viscosity ration for the $\mathcal{N} = 4$ SYM plasma (Myers et.al):

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \frac{5}{16}\,\frac{\lambda^{1/2}}{N^2} + \cdots\right)$$

$\Longrightarrow$ Notice that the KSS viscosity bound survives — does it mean that it is always true in holographic plasma?

NO! — finite $\frac{1}{N}$ corrections

$\Longrightarrow$ A given conformal gauge theory is characterized by two different central charges c and a, defining its conformal anomaly

$$\langle T^{\mu}_{\mu} \rangle = \frac{c}{16\pi^2} I_4 - \frac{a}{16\pi^2} E_4$$

where

$$E_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \,, \qquad I_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$\Longrightarrow$ In the planar limit

$$c = a$$

$\Longrightarrow$ In a conformal toy model of QCD we expect

$$c \neq a$$

because of the presence of fundamental matter.

Consider an effective higher-derivative model of gauge theory/string theory duality

$$S = \int d^5x\sqrt{-g}\bigg(\frac{1}{\kappa^2}R - \Lambda + c_1 R_{abcd}R^{abcd} + c_2 R_{ab}R^{ab} + c_3 R^2 + \mathcal{O}(R^4)\bigg)$$

where $\kappa^2 = 16\pi G_N$. The holographic conformal anomaly is

$$\langle T^\mu_\mu\rangle_{holographic} = \left(-\frac{l^3}{8\kappa^2} + c_2 l + 5c_3 l\right)(E_4 - I_4) + \frac{c_1 l}{2}(E_4 + I_4)$$

while Kats et.al and Brigante et.al found

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{8c_1\kappa^2}{\ell^2} + \cdots\right) = \frac{1}{4\pi}\left(1 - \frac{(c-a)}{c} + \cdots\right) = \frac{1}{4\pi}(1 - \Delta + \cdots)$$

- Notice that c_1 coefficient can come only form $R_{abcd}R^{abcd}$, and it is precisely the coefficient that corresponds to having in the dual CFT $c \neq a$. In particular R^4-terms does not effect $(c-a)$ anomaly of a CFT.
- The KSS viscosity bound is violated in a CFT whenever (c-a). The violation is under control, if $|c-a|/c \ll 1$.

Non-universal violation of the KSS bound

Consider a superconformal gauge theory. The superconformal algebra implies the existence of an anomaly-free $U(1)_R$ symmetry. It was found in Anselmi et.al that

$$c - a = -\frac{1}{16}\left(\dim G + \sum_i (\dim R_i)\,(r_i - 1)\right)$$

$$c = \frac{1}{32}\left(4\,(\dim G) + \sum_i (\dim R_i)\,(1 - r_i)\left(5 - 9(1 - r_i)^2\right)\right)$$

where r_i denote the R-charge of a matter chiral multiplet in the representation R_i

$\Longrightarrow$ So all we need to do is to scan through the list of available CFT's and compute $(c - a)$.

- Superconformal gauge theories with exactly marginal gauge coupling

Consider $SU(N_c)$ supersymmetric gauge theory with n_{adj} χsf in the adjoint representation, n_f flavors in the fundamental representation, n_{sym} flavors in the symmetric representation and n_{asym} flavors in the anti-symmetric representation. It is easy now to enumerate all the models with $G = SU(N_c)$ and $\Delta \ll 1$ as $N_c \to \infty$:

	$(n_{adj}, n_{asym}, n_{sym}, n_f)$	$c - a$	Δ
(a)	(3,0,0,0)	0	0
(b)	(2,1,0,1)	$\frac{3N_c+1}{48}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$
(c)	(1,2,0,2)	$\frac{3N_c+1}{24}$	$\frac{1}{2N_c} + \mathcal{O}(N_c^{-2})$
(d)	(1,1,1,0)	$\frac{1}{24}$	$\frac{1}{6N_c^2} + \mathcal{O}(N_c^{-4})$
(e)	(0,3,0,3)	$\frac{3N_c+1}{16}$	$\frac{3}{4N_c} + \mathcal{O}(N_c^{-2})$
(f)	(0,2,1,1)	$\frac{N_c+1}{16}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$

For the $Sp(2N_c)$ supersymmetric gauge theories

	(n_{adj}, n_{asym}, n_f)	$c-a$	Δ
(a)	(3,0,0)	0	0
(b)	(2,1,4)	$\frac{6N_c-1}{48}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$
(c)	(1,2,8)	$\frac{6N_c-1}{24}$	$\frac{1}{2N_c} + \mathcal{O}(N_c^{-2})$
(d)	(0,3,12)	$\frac{6N_c-1}{16}$	$\frac{3}{4N_c} + \mathcal{O}(N_c^{-2})$

$\Longrightarrow$ The are no models in this class with orthogonal gauge groups

- $\mathcal{N}=2$ superconformal fixed points from F-theory

Consider N D3-branes probing an F-theory singularity generated by n_7 coincident (p,q) 7-branes, resulting in a constant dilaton. As $N\to\infty$,

$$c-a=\frac{1}{4}N\,(\delta-1)-\frac{1}{24}\,, \qquad \Delta=\frac{\delta-1}{N\delta}+\mathcal{O}(N^{-2})$$

where δ is a definite angle characterizing an F-theory singularity with a symmetry group $\mathcal{G}$

$\mathcal{G}$	H_0	H_1	H_2	D_4	E_6	E_7	E_8
n_7	2	3	4	6	8	9	10
δ	6/5	4/3	3/2	2	3	4	6

Notice that in all cases $0<\Delta\ll 1$ as $N\to\infty$.

$\Longrightarrow$ In all examples presented the KSS bound is violated since $(c - a) > 0$

$\Longrightarrow$ There many more CFT's with $c \neq a$. For them, however, $c - a \sim c$ and so we can not say anything reliable about KSS bound. Curiously though, we did not find a single CFT with $c \neq a$ so that $(c - a) < 0$ — in particular there also no models of this type involving the product gauge groups.

Is there a bound on $\frac{\eta}{s}$?

$\Longrightarrow$ In realistic holographic models (derivable from string theory) corrections to η/s are due to higher derivative terms, thus these corrections are necessarily perturbative.

$\Longrightarrow$ To address the bound on η/s one considers *models* of AdS/CFT correspondence:

- we use the rules of holography
- we do not care whether or not the model is embeddable in string theory

$\Longrightarrow$ A nice model of this type is a Gauss-Bonnet gravity:

$$\mathcal{I} = \frac{1}{2\ell_P^3} \int d^5x \sqrt{-g} \left[\frac{12}{L^2} + R + \frac{\lambda_{GB}}{2} L^2 \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right) \right]$$

This model is solvable for any λ_{GB} *i.e.,* , we can find exact (analytic) black hole solution and study its near-equilibrium properties.

$\Longrightarrow$Computing the boundary stress-energy tensor of $\mathcal{I}$ we identify a dual gauge theory as a CFT with central charges $\{c, a\}$ given by

$$c = \frac{\pi^2}{2^{3/2}} \frac{L^3}{\ell_P^3} (1 + \sqrt{1 - 4\lambda_{GB}})^{3/2} \sqrt{1 - 4\lambda_{GB}}$$

$$a = \frac{\pi^2}{2^{3/2}} \frac{L^3}{\ell_P^3} (1 + \sqrt{1 - 4\lambda_{GB}})^{3/2} \left(3\sqrt{1 - 4\lambda_{GB}} - 2\right)$$

or

$$\frac{c - a}{c} = 2 \left(\frac{1}{\sqrt{1 - 4\lambda_{GB}}} - 1 \right)$$

$\Longrightarrow$ It is straightforward to study dispersion relation of the quasinormal modes of the GB black holes — these quasinormal modes are dual to linearized fluctuations in plasma (the shear and the sound channel modes)

For the shear viscosity one finds (Brigante *et.al*)

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[1 - 4\lambda_{GB} \right]$$

$\Longrightarrow$ Note that result is exact in λ_{GB}. Thus constraining the possible values of λ_{GB} would put constraint on $\frac{\eta}{s}$

$\Longrightarrow$ I should say that I believe the necessity to come up with the physics to 'constrain' λ_{GB} arises because the model is phenomenological, rather than a String Theory

$\Longrightarrow$ I explain now how causal second-order hydrodynamics restricts λ_{GB}.

Causality constraints on the transport coefficients of the hydrodynamics

Hydrodynamics is an effective theory describing near-equilibrium phenomena in (relativistic) QFT:

$$\nabla_\nu T^{\mu\nu} = 0$$

The stress-energy tensor includes both an equilibrium part (ϵ and P terms) and a dissipative part $\Pi^{\mu\nu}$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \Pi^{\mu\nu} \,.$$

where u^μ is a local 4-velocity of the fluid and

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu \,, \quad \Pi^\mu{}_\nu u^\nu = 0 \,, \quad u^\mu \nu_\mu = 0 \,,$$

Effective hydrodynamic description is equivalent to a derivative expansion of $\Pi^{\mu\nu}$ in local velocity gradients

Thus, to linear order in the derivative expansion

$$\Pi^{\mu\nu} = \Pi_1^{\mu\nu}(\eta, \zeta) = -\eta\sigma^{\mu\nu} - \zeta\Delta^{\mu\nu}(\nabla_\alpha u^\alpha)$$

$(\sigma^{\mu\nu} \propto \nabla_\nu u^\mu)$ with $\{\eta, \zeta\}$ being the viscosity coefficients.

To simplify further discussion we consider only CFT's from now on: $\zeta = 0\,, \epsilon = 3P$. To second order in the derivative expansion

$$\Pi^{\mu\nu} = \Pi_1^{\mu\nu}(\eta) + \Pi_2^{\mu\nu}(\eta, \tau_\Pi, \kappa, \lambda_1, \lambda_2, \lambda_3)$$

$$= -\eta\sigma^{\mu\nu} - \eta\tau_\Pi\left[\langle u \cdot \nabla\sigma^{\mu\nu}\rangle + \frac{1}{3}\left(\nabla \cdot u\right)\sigma^{\mu\nu}\right] + \mathrm{non-linear\ terms} + \cdots$$

$\Longrightarrow$ It is straightforward to study dispersion relation of the linearized fluctuations in above theory

The dispersion relation of the shear channel fluctuations is given by

$$0 = -\mathfrak{w}^2 \ \tau_\Pi T - \frac{i\mathfrak{w}}{2\pi} + \mathfrak{q}^2 \ \frac{\eta}{s} \,,$$

where $\mathfrak{w} = \omega/(2\pi T)$ and $\mathfrak{q} = k/(2\pi T)$. Now the speed with which a wave-front propagates out from a discontinuity in any initial data is governed by

$$\lim_{|\mathfrak{q}|\to\infty} \ \frac{\mathrm{Re}\,(\mathfrak{w})}{\mathfrak{q}}\bigg|_{[\mathrm{shear}]} = \sqrt{\frac{\eta}{s \ \tau_\Pi T}} \equiv v^{front}_{[\mathrm{shear}]} \,.$$

Hence causality in this channel imposes the restriction

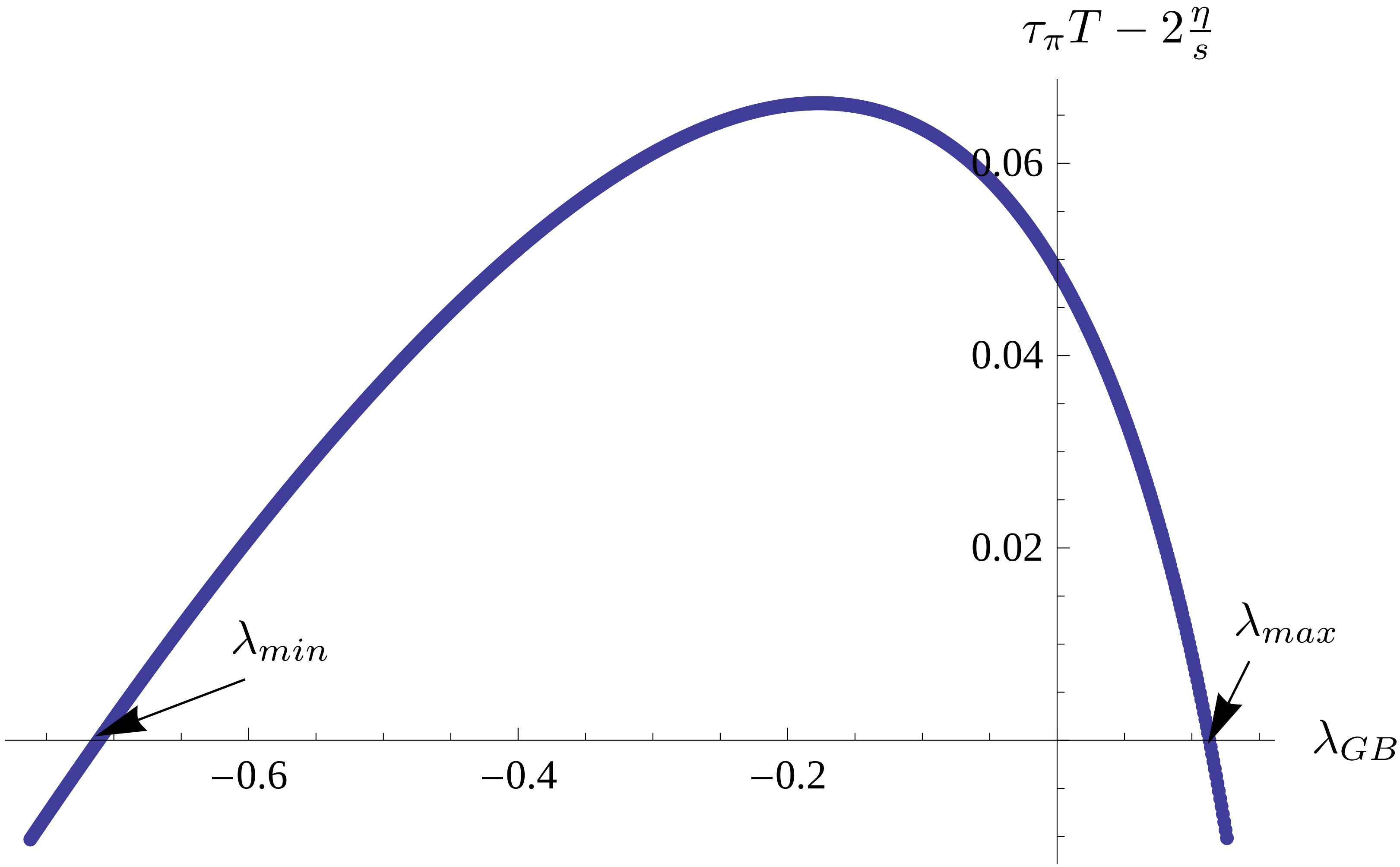
$$\tau_\Pi T \ \geq \frac{\eta}{s} \,.$$

Notice: the first-order hydrodynamics is recovered in the limit $\tau_\Pi \to 0$, so causality is always violated at this order in the derivative truncation

Similar considerations in the sound channel imposes the (more stringent) restriction

$$\tau_\Pi T \ \geq 2\frac{\eta}{s} \,.$$

For the hydrodynamic relaxation time in Gauss-Bonnet gravity:



Causality of the second-order Gauss-Bonnet hydrodynamics is violated once $\tau_\Pi T < 2\frac{\eta}{s}$. Thus, $\lambda_{GB} \in [\lambda_{min}, \lambda_{max}]$, where $\lambda_{min} = -0.711(2)$ and $\lambda_{max} = 0.113(0)$.

$\Longrightarrow$ Previous analysis were based on the effective theory of the successive local-velocity derivative expansion in GB plasma

$\Longrightarrow$ However, given the gravity dual we can study dispersion relation of quasinormal modes in GB plasma without any reference to a hydrodynamic expansion!

In this way we find that causality is violated in GB CFT plasma, unless

$$-\frac{7}{36} \le \lambda_{GB} \le \frac{9}{100}$$

which translates into the following constrain on the CFT central charges

$$-\frac{1}{2} < \frac{c-a}{c} < \frac{1}{2}$$

and correspondingly, introduces the lower bound on the shear viscosity

$$\frac{\eta}{s} \ \ge \ \frac{1}{4\pi}\,\frac{16}{25}$$

$\Longrightarrow$ We could have phrased the discussion above in a different context:

■ As I show in a bit, uncertainty relation of QM implies that there *must* exist a bound of the type

$$\frac{\eta}{s} \gtrsim \frac{\hbar}{k_B}$$

■ AdS/CFT (in a regime when it is tractable) maps QFT into a classical gravity, so above relation *must* be enforced in it!

■ We could have used GB gravity and try to violate the viscosity bound — we found that in doing this, we necessarily have to violate causality.

So, did we actually solve the problem?, i.e, did we establish that trying to violate uncertainty relation in QM necessitates the violation of causality in a holographic setting?

unfortunately....

I believe, NO

in fact,

$\Longrightarrow$ I will show that there is a holographic model which circumvents the causality problem while trying to bring

$$\frac{\eta}{s} \to 0$$

$\Longrightarrow$ What breaks down in that model? (How QM is enforced in this more involved holographic example?)

A QM bound on shear viscosity

$\Longrightarrow$ Assume a quasiparticle picture of the plasma fluid.

- From kinetic theory, in relativistic plasma

$$\eta \sim \epsilon \tau_{mfp} \, , \qquad s \sim k_B n$$

where ϵ is the energy density, s is the entropy density, n is the number density of quasiparticles, and τ_{mfp} is the mean free path.

- So

$$\frac{\eta}{s} \sim \frac{1}{k_B} \times \left(\frac{\epsilon}{n} \qquad \times \qquad \tau_{mfp} \right) \qquad \gtrsim \frac{1}{k_B} \times \hbar$$

$$\qquad\qquad \Uparrow \qquad\qquad\qquad \Uparrow$$

from QM uncertainty principle:

$$\left(\Delta E \qquad \times \qquad \Delta t \right) \qquad \gtrsim \hbar$$

$\Longrightarrow$ Heuristically, the reason why in previous GB gravity model the causality (a UV property of the model) was linked to the shear viscosity (an IR, hydrodynamic, property of the theory) was because there was **no phase transition as one goes from UV to the IR**

$\Longrightarrow$ Thus, to break above link, all one has to go is to consider a model with a continuous phase transition.

$$\Downarrow$$

$\Longrightarrow$ more precisely, we need continuous holographic phase transition in the presence of higher derivative terms, since the universality of holographic shear viscosity guarantees that

$$\frac{\eta}{s} = \frac{\hbar}{k_B} \qquad \times \qquad \frac{1}{4\pi}$$

even **in the presence of the phase transition**

The model

- before:

$$\mathcal{I} = \frac{1}{2\ell_P^3}\int d^5x\sqrt{-g}\left[\frac{12}{L^2} + R + \frac{\lambda_{GB}}{2}L^2\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)\right]$$

$$= \frac{1}{2\ell_P^3}\int d^5x\sqrt{-g}\left[\mathcal{L}_E(g_{\alpha\beta}) + \frac{\lambda_{GB}}{2}\times GB\right]$$

- now

$$\mathcal{I} = \frac{1}{2\ell_P^3}\int d^5x\sqrt{-g}\left[\mathcal{L}_{superconductor}(g_{\alpha\beta}; A_\mu, \phi) + g\phi^4 \times GB\right]$$

where g is a coupling constant; ϕ is a charged scalar field that condenses in the IR (low temperatures).

$\Longrightarrow$ The condensed scalar field plays the role of the effective GB coupling:

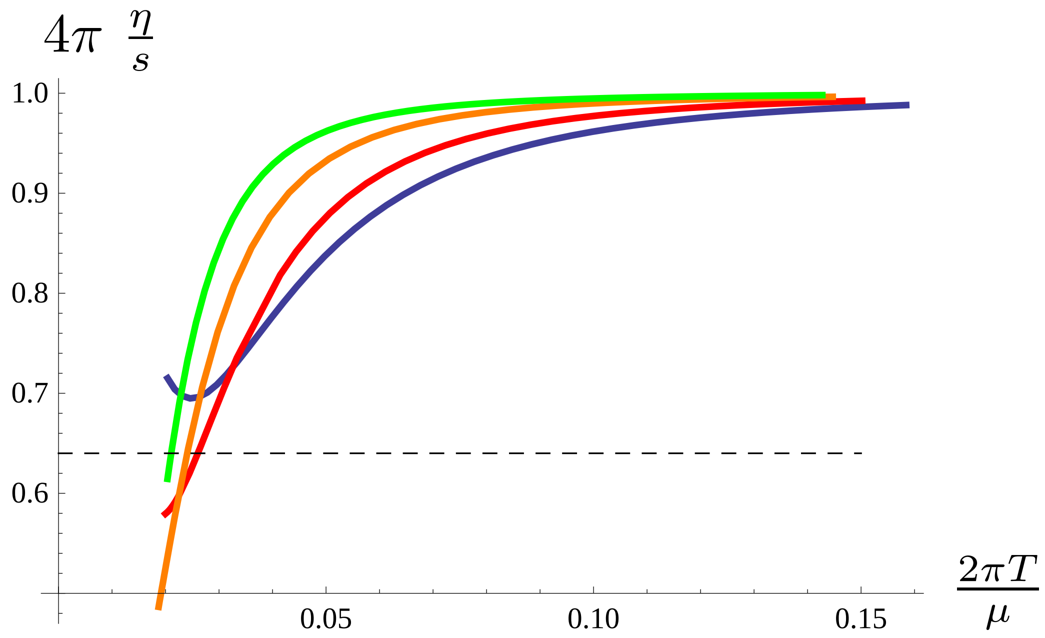
$$g\phi^4\Big|_{horizon} = \frac{\lambda_{GB}}{2}$$

which follows from the membrane paradigm of the black hole hydrodynamics

$\Longrightarrow$ however, this scalar field **can not** modify the causality property of the boundary QFT — at a technical level, the term

$$g\phi^4 \times GB$$

will not contribute to the boundary stress-energy tensor, because of the boundary fall-off of ϕ.



Ratios of shear viscosity to entropy density in the broken phase for select values of the coupling: $g = -1$ (blue), $g = -2$ (red), $g = -5$ (orange) and $g = -10$ (green), as a function of $\frac{2\pi T}{\mu}$. The dashed black line indicates the Gauss-Bonnet viscosity bound: $\eta/s \geq 16/25$.

How QM rules in this model?

Summary on corrections to shear viscosity in holographic conformal models

■ There are generically 2 types of corrections:

- finite (exactly) marginal coupling corrections — t' Hooft coupling corrections
- nonplanar corrections due to $(c - a) \neq 0$

■ The former (universally) satisfy KSS bound, while the latter (universally) violate it

■ In some simple holographic models there is a lower bound on η/s, induced by the violation of the microcausality in the theory

■ It is possible to engineer (seemingly consistent) holographic models with arbitrarily low η/s